Аз съм Джим Bezdek















Sockeye Salmon, Seattle, 1983



Extending fuzzy and probabilistic clustering to VL data sets, <i>Comp. Stat. And Analysis</i> , 2006.	Fuzzy c-Means Algorithms for Very Large Data, IEEE TFS, 2012.
Approximate clustering in very large relational data, IJIS, 2006.	A hybrid approach to clustering in big data, <i>IEEE</i> <i>Trans. Cybernetics,</i> 2016.

2 Kinds of *Basic Numerical Data* for Pattern Recognition

Objects	O= {o ₁ ,, o _n } : o _i = i-th <i>physical</i> object
Object Data	$\begin{array}{llllllllllllllllllllllllllllllllllll$
	Sizes : n = # samples; p = # dimensions
Relational Data	R = [r _{ij}] = <i>relationship</i> (o _i , o _j)) or (x _i , x _j) s _{ij} = pairwise <i>similarity</i> (o _i , o _j) or (x _i , x _j) d _{ij} = pairwise <i>dissimilarity</i> (o _i , o _j) or (x _i , x _j)
Typically (R = D)	$d_{ii} = 0 : 1 \le i \le n$ $d_{ij} > 0 : 1 \le i, j \le n$ (Positive-definite) $d_{ij} = d_{ji} : 1 \le i \ne j \le n$ (Symmetric)
We	often convert $X \rightarrow D$ with <i>distance</i> $d_{ij} = x_i - x_j $



How Big is static BIG Data ?



Bytes	<i>Big Data</i> (BD) in size (n, p)
10 ³	Kilo – small
106	Mega – medium
10 ⁹	Giga – large
1012	Terra – getting' up there
1015	Peta - MONSTER
10>18	Exa -Yikes ! (BIG DATA)
We can't clus	ster or image data this big (in a single computer)
Mos	t BIG data methods build "cluster-friendly" loadable) subsets by <i>sampling</i> or <i>chunking</i>

Clustering in Big Data

SO

4 ways to make static Big (BD_N) Data Small(er)



Four Data Levels



Case 1	If X _n or D _n is <i>Loadable</i> we can <i>Quantitatively Compare</i> Lit-Clusters ↔ Approx. Clusters
Case 2	If X _N or D _N is <i>Unloadable</i> <i>Comparison is impossible</i>
SO	case 2 validity rests with "good" case 1 examples

Case 2 BD methods often provide acceleration and feasibility

Case 2 If X_N or D_N is Unloadable ... there are 3 basic approaches to scaling up

1 Cluster a sample, non-iterative extension

Incremental/Distributed data clustering

3 Kernel-based methods, e.g., kFCM

Many of these methods can be used with other pattern recognition algorithms. For clustering, we let ...

 $A^* = [U^*, V^*] = exact$ (literal) partition and prototypes A = [U, V] = (any) approximation to A^* by (1) or (2)

2



Partition Matrices

Membership Functions

u_i:**O→**[0,1]

 $u_i(o_k)=u_{ik}=M'ship$ of o_k in cluster i

col k 🖛 M'ship of o_k in each cluster

	Crisp		Fuzzy/Prob		Possibilistic
Row sums	$\sum_{k} u_{ik} > 0$		same		same
Col sums	$\sum_{i} u_{ik} = 1$		same		∑ ^c u _{ik} ≤ C i=1
M'ships	$\mathbf{u_{ik}} \in \{0,1\}$	•	<mark>u_{ik} ∈ [0, 1]</mark>		same
Set Name	M _{hcn}	\subset	M _{fcn}	\subset	M _{pcn}
Example	1 0 0 0 0 1 0 0 0 0 1 1		1 .07 0.44 0 .91 0.06 0 .02 1.50		1 .07 1 .44 0 .91 0 .52 0 .02 1 .38
					Δ

Batch Hard and Fuzzy c-Means Models

Objective function

Inputs

$$\mathbf{J}_{m}(\mathbf{U},\mathbf{V}) = \sum_{k=1i=1}^{n} \sum_{i=1}^{c} \mathbf{u}_{ik}^{m} \|\mathbf{x}_{k} - \mathbf{v}_{i}\|_{A}^{2}$$
$$\|\mathbf{x}_{k} - \mathbf{v}_{i}\|_{A}^{2} = (\mathbf{x}_{k} - \mathbf{v}_{i})^{T} \mathbf{A} (\mathbf{x}_{k} - \mathbf{v}_{i})$$

Object data $X = \{x_1, \dots, x_n\} \subset \Re^p$



Optimization Problem, m≥1

$$\underset{\substack{\boldsymbol{U} \in \boldsymbol{M}_{fcn} \\ \boldsymbol{V} \in \boldsymbol{\Re}^{cp}}}{\text{minimize}} \left\{ \boldsymbol{J}_{m}(\boldsymbol{U}, \boldsymbol{V}) = \sum u_{ik}^{m} \left\| \boldsymbol{x}_{k} - \boldsymbol{v}_{i} \right\|_{\boldsymbol{A}}^{2} \right\}$$

FONCs for extrema of the HCM/FCM Functionals



Weighted FCM/HCM = wFCM/wHCM





What is *Progressive Sampling*?





Sample
$$X_{5} \subset X$$

(Non-Iterative) Generalized extension of *Fuzzy c-Means* [FCM \rightarrow eFFCM/geFFCM]
Note: also works for HCM
Extend FCM[X_{5}] \rightarrow FCM[X-X_{5}] with prototypes V₅ and $x_{k} \in X-X_{5}$
Remark: The extension step usually takes ~ 1% of overall CPU time
 $(X - X_{5}) \ni x_{k} \{ \cdots v_{i,5} \cdots v_{j,5} \cdots \} = V_{5}$
 $u_{ik} = \left[\sum_{j=1}^{c} (\|x_{k} - v_{i,5}\|_{A} / \|x_{k} - v_{j,5}\|_{A})^{\frac{2}{m-1}} \right]^{-1} = \varphi(V_{5}, X - X_{5})$
FONC for
U to
min J_m Works as a classifier on X-X₅, but trained w^o labels !



Input image LFCM 100% of data 256x256x256 mrFCM 100% of data eFFCM **eFFCM** 9% of data, div only **29%** of data, div & χ2

Example: PS + E with FCM on an

Indian Satellite (Very Small Landsat Image)

Typical Output Images

COMPARE 9% vs 100% !

Incremental/Distributed Clustering in **BIG** Data



3 Problems for the Distributed Clustering Approach

Incremental/Distributed c-Means Clustering in VL Data

brFCM	= "bit	reduct."	FCM
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Eschrich/Ke/Hall/Goldgof (2003, brFCM). Fast accurate fuzzy clustering through data reduction. IEEE TFS, 11(2), 262-270.

Compression for image data, uses wFCM algorithm, (loadable) implementation

spFCM =	single pass" FCM	Hore, Hall, Goldgof (<i>2007, spFCM</i>). Single pass fuzzy c- means, Proc. FUZZ-IEEE 2007, 1-7.
	Uses wFCM algorithm,	partially distributed VL implementation

oFCM = "on line" FCM	Hore/Hall/Goldgof/Gu/Maudsley/Darkazanli (<i>2009, oFCM</i>). A scalable framework for segmenting MRIs, <i>J Signal Proc. Syst</i> . 54(1-3), 183-203.
Uses wFCM	algorithm, fully distributed VL implementation

All 3 are generalizations of HCM (k-means) when m = 1

spFCM="single pass" and oFCM = "on line" FCM

Split data
$$X = \bigcup X_j$$
 : $n = \sum n_j$ First pass $FCM(X_1) = (\bigcup, V)$ Rowsums of \bigcup
after pass $j \ge 1$ $\omega_i = \sum_{k=1}^{n_j} u_{ik}; 1 \le i \le c$ weights for wFCM
before pass $j > 1$ $w = ([1], \Omega) = (1, 1, ..., 1, \omega_1, ..., \omega_c)$

Architecture of spFCM : c is chosen and fixed by user



Architecture of oFCM : c = "max" is same for all blocks



Visual comparison of segmentation with spfcm/ofcm to EM



Comparing rseFCM, spFCM & oFCM to LFCM

rseFCM = random sampling + LFCM(X_{ns}) + extension to $X_N - X_{ns}$

LFCM = literal FCM (loadable data) + extension to $X_N - X_{ns}$

spFCM = single pass FCM (Hall's model) + extension to X_N-X_{ns}

oFCM = online FCM (Hall's model) + extension to $X_N - X_{ns}$



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MNIST Data: n=70,000, p=784, c=10

Each Image: 28 x 28

Each Pixel p_{ii}: 0 to 255

Each Pixel Normalized: (p_{ii}/255) so 0 to 1

Each Vector: 784 values $p = (p_{11}, ..., p_{ij}, ..., p_{28,28})$

Presumably c = 10, but ... this data does *NOT* cluster well

MNIST Data n=70,000, p=784, c=10 (Typical Results)



LFCM line (----) is ARI match of $H(U_{LFCM})$ to U_{GT}

Other graphs show ARI matching of $H(U_{VL})$ to U_{GT}

Approximation quality of VL-FCMs: compare other graphs to LFCM



CLUSTERING IN VL DATA

Forest Data n=581,012, p=54, c=7



MRI Image Data n ~ 4×10^6 , p=1 or 3, c=3

	0.3	1% sam	ples		1% samples			10% samples		
p=1	SU	ARI ₂	ARI _s	S	J ARI ₂	ARI		SU	ARI ₂	ARI _s
rseFCM	22	0.97	0.66	18	3 0.99	0.66		7	1	0.66
spFCM	13	0.98	0.66	13	0.98	0.66		8	0.98	0.66
oFCM	2	1	0.66	4	1	0.66		4	1	0.66
brFCM	108	1	0.66	50) 1	0.66		8	1	0.66
				_						
p=3	SU	ARI ₂	ARI _s	SI	J ARI ₂	ARI		SU	ARI ₂	ARI _s
rseFCM	29	0.97	0.47	24	4 1	0.47		8	1	0.47
spFCM	18	0.96	0.46	13	3 0.96	0.46		7	0.96	0.46
oFCM	2	0.78	0.38	2	0.93	0.44		3	1	0.47

SU = "speed up" : $ARI_2(H(U), H(U_{LFCM}))$: $ARI_s(U, U_{LFCM})$)



Input $D_{N\times N} : D_{ii} \ge 0 ; D_{ii} = 0 : D = D^{T}$

sVAT/siVAT with maximin sampling for BIG data

Initialize

c' \geq c An (OVER) estimate of c n \leq 6000 Approximate sample size m₁ = 1 (arbitrary) o₁ = 1st Prototype (Index) d = (d₁,...d_N) = (D₁₁,...D_{1N}) search array Get c' Maximin Samples; i.e., Get indices {m_i} of (c') prototypes = {o_{m1},...,o_{mk},...,o_{mc1}}

What are Maximin Samples?



Get crisp 1-np (HCM) clusters of $\{o_{m_k}\}$ - (may need paging)







A second approach: *e spec VAT* for VL data



Wang/Geng/Bezdek/Leckie/Kotagiri, (2010). spec-VAT for cluster analysis, IEEE TKDE.



n = 100 random samples from the 20 partitions



CLUSTERING IN BIG DATA

clusiVAT image of the n = 100 samples implies c = 10









Comparing 5 *crisp* BD clustering algorithms: clusiVAT, HCM="*k-means*", spHCM, oHCM, CURE

Time : CPU time in secs

Partition Accuracy of crisp U

$$\mathsf{PA}(\mathsf{U}|\mathsf{U}_{\mathsf{GT}}) = \frac{\langle \mathsf{U}_{\mathsf{GT}}, \mathsf{U} \rangle}{\mathsf{n}} = \frac{\sum_{i=1}^{\mathsf{c}} \mathsf{n}_{i}}{\mathsf{n}} = \left(\frac{\texttt{\# matched}}{\texttt{\# tried}}\right)$$

 U_{GT} = "ground truth" partition of crisp labels

25 run averages for 12 small sets of CS Gaussian Clusters

Data-s	et Informat	ion	clusiVAT		k-means		ans		
Total	Clusters	DI	Accuracy	Time (s)	Ac	euracy	Time (s)		
No. of			(%)		(%))			
points									
1000	3	1.199	100	0.030	- 76	6.264	0.013		
1000	4	1.037	100	0.059	- 79	9.310	0.023		
1000	5	1.039	Single p	ass k-means	2	Online	k-means	CU	RE
2000	3	1.006	Accuracy	Time (s)		Accuracy	Time (s)	Accuracy	Time (s)
2000	4	1.195	(%)			(%)	11110 (5)	(%)	11110 (5)
2000	5	1.030				()		()	
5000	3	1.036	85.802	0.023		70.769	0.018	100	11.236
5000	4	1.175	99.071	0.032		62.087	0.020	100	10.925
5000	5	1.075	81.566	0.029		55.265	0.016	100	10.722
10,000	3	1.181	100	0.022		85.835	0.021	100	11.040
10,000	4	1.121	100	0.029		78.709	0.028	100	10.921
10,000	5	1.120	66.865	0.088		45.888	0.030	100	10.823
Ave	rage value	s	100	0.067		84.811	0.051	100	11.344
			100	0.053		67.472	0.057	100	11.098
			97.850	0.062		77.291	0.062	100	10.853
			100	0.086		94.983	0.098	100	11.395
			100	0.094		73.034	0.102	100	11.284
			100	0.094		73.901	0.108	100	11.110
			94.263	0.057		72.504	0.051	100	11.063

Mean averages for 12 BIG sets of CS Gaussian Clusters. Ave. Size N = 450,000

	c'iVAT	h-km	sp-hkm	ol-hkm	CURE	
large CS	0.977	4.487	3.934	3.834	31.30	time, secs
	100	72.77	99.67	70.19	99.81	accuracy,%

Mean averages for 12 BIG sets of non-CS Gaussian Clusters. Ave. Size N = 450,000

	c'iVAT	h-km	sp-hkm	ol-hkm	CURE	
large NCS	1.021	4.395	5.163	4.680	31.04	time, secs
	99.99	75.14	90.99	73.93	97.83	accuracy,%

clusiVAT is fastest AND most accurate







10 continuous features 40 binary soil types 4 binary wilderness types



	c'iVAT	h-km	sp-hkm	ol-hkm	CURE	
Forest	4.049	46.53	59.24	173.5	59.48	time, secs
	43.7	11	15	4	43.6	accuracy,%

KDD-99 Cup data: (22 simulated attacks + normal data) → c = 23

41 features in [0, 1] N = 4,292,637 c = 23 class labels siVAT image for n = 230
4 (major) attack types
Denial of Service (DOS) Users to Root (U2R) Remote to Local (R2L) Probing Attacks (PROBE)

c'iVAT	hkm	spkm	olkm	CURE	
97.06	94.25	96.45	94.87	91.54	accuracy,%
76.0	124.8	120.4	138.5	841.6	time, secs

A few acceleration schemes for literal algorithm \mathcal{A}

Ref.	A	P	с	n	speedup
Arthur	НСМ	5-35	5-50	10,000,0.5M	1-9.6:1
Hore	НСМ	3-617	3-12	150, 4M	600,000:1
Pelleg	НСМ	2	5000	0.4M	26-136:1
Moore	EM	2-6	5,320	12,500,0.7M	9-500:1
Thiesson	EM	2, 33	303,12	21,888,0.6M	1.7-2.8:1
Ortiz	EM	2	2	2,000	1-12:1
March	SL	3,3840	10, v	4(10⁴)-10⁶	3:1
Müllner	SL	2, 100	1, 5	10,10,000	10:1

A few acceleration schemes for $\mathcal{A} = fuzzy \ c-means$

Ref.	\mathcal{A}	Р	С	n	speedup
Cannon	FCM	10	10	0.25 mb	6:1
Kamel	FCM	v	v	Small	1.2:1
Cheng	FCM	3, 6	10	0.4 mb	3:1
Altman	FCM	3	3	1 mb	3-10:1
Kolen	FCM	9	10	20 mb	9:1
Borgelt	FCM	8-13	2, 3	<u>≺</u> 4177	2:1
Anderson	FCM	4-32	4-64	64,8192	10-100:1
Eschrich	FCM	2,3	5,7	0.4mb	59-290:1

.

Empirical Conclusions: pseFCM & rseFCM



Three types (random, progressive, Maximin). Easily adaptable for extensions to Big Data with *many* other algorithms



rseFCM

Non-iterative scaling for *many* algorithms typically incurs about 1% of total CPU time

Superiority to pseFCM increases with n

Faster than spFCM/oFCM for large n

Average speedup of LFCM ~ 30:1

Good Approximation to LFCM clusters

Empirical Conclusions: brFCM, spFCM & oFCM



Recommendations: Big Data fuzzy c-Means AND its special case, HCM = "k-means" at m=1



Empirical Conclusions: siVAT and clusiVAT

ClusiVAT works (so far !)

() the siVAT image usefully estimates c *before* clustering

() is EXACT (scalable) SL when DI > 1

 $(\stackrel{()}{=})$ is much more accurate than batch and incremental k-means

is 25-250 times *faster* than CURE

Things to fix and do

(1) SL can go awry if data is very "stringy"

() Next up: incremental clusiVAT for streaming data !

What Happens Next?

"Data-driven decisionmaking is another sign that the role of the campaign pros in Washington who make decisions on hunches and experience is rapidly dwindling, being replaced by quants and computer coders who can crack massive data sets for insight. As one official put it, the time of "guys sitting in a back room smoking cigars, saying 'We always buy 60 Minutes'" is over. In politics, the era of big data has arrived."

M. Scherer, Inside the Secret World of Quants and Data Crunchers who helped ObamaWin, *Time Magazine*, Nov. 19, 2012, 56-60.





2 ONSLAUGHT OF BIG DATA BUZZWORDS

[7 Vs: volume, velocity, veracity, value, variety, validity, value !!!]

9/7/16



Clustering in Big Data









With these aids my hearing is about 8% of normal





I will try to answer questions, but a better result follows if you email them to me.

Questions, pdf's of today's talk and papers

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